

# Solutions of Bethe-Salpeter equation in QED<sub>3</sub>

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## Abstract

To understand the mechanism of the fermion pair and fermion-antifermion pair condensation, the solutions of Bethe-Salpeter equation in QED<sub>3</sub> is examined. In the ladder approximation our solution for the axial-scalar is consistent with Ward-Takahashi-identity for the axial vector currents. Since the massless scalar-vector sector is described by a coupled integral equation, it is difficult to solve explicitly. We approximate the equation for large and small momentum region separately and convert them into differential equations in position space. These equation can be solved easily. Boundary condition at the origin leads the eigenvalue for dimensionless coupling constant  $\lambda = e^2/m$ . There exists solutions for massless scalar-vector fermion-antifermion (fa) system with discrete spectrum. In our approximation massless-scalar-vector ff system does not seem to exist.

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## I. INTRODUCTION

In the theory of superconductivity and superfluidity BCS model is very familiar and useful to analyse their properties[1].If we assume the  $\delta^{(3)}(x - y)$  function type interaction between fermions it is not difficult to solve the gap equation and its dependence of the coupling constant.However we have not been known the reason why the electron pair form a bound state.On the other hand there exists solutions of Bethe-Salpeter equation in the ladder approximation for fermion-fermion pair with discrete spectrum[2].Recently a bound state for quasi particles(exiton) are considered in terms of approximate Bethe-Salpeter(BS) equation in QED<sub>3</sub> for phase fluctuating d-wave superconductor near the node to measure the resonant spin response[3].The Schrödinger type equation for particle-hole boundstate with potential  $1/r^2$  was derived and the eigenvalue condition emerged.Its solution indicates the existence of strong coupling phase but seems to be not normalizable.Therefore it is interesting to apply relativistic BS equation for fermion pair in the same approximation in[2].We think that it is important to solve the equation for the massless boundstate which signals the instability of the vacuum under condensation of these bosons in field theoretical model[4].In this work first we examine the existence of solutions for the massless boundstates which are normalizable.Since scalar-vector sector is written by coupled integral equation,it is difficult to solve the equation explicitly.Therefore we approximate the equation for the large and

small momentum region separately. These equations are easily solved in position space and we have a correct short and long-distance behaviour. For a system the boundary condition at the origin leads dimensionless coupling constant  $\sqrt{2}e^2/4\pi m = \text{integer}$  where  $m$  a fermion mass. In section II we introduce spinor-spinor BS equation and show their solutions in QED and QED<sub>3</sub>[2]. In section III we compare the solutions of Dyson-Schwinger equation for the fermion propagator with axialscalar solutions of BS equation. In our approximation they obey the same equation and Ward-Takahashi-identity for axial currents is satisfied. Section IV is devoted for summary.

## II. SPINOR-SPINOR BS EQUATION

### A. Massless boson in QED

BS amplitude in four-dimension is defined[6,7,8]

$$\chi(x_1 x_2; B) \equiv \langle 0 | T(\psi(x_1) \bar{\psi}(x_2)) | B \rangle, \quad (1)$$

for the fermion-antifermion bound state  $|B\rangle = |f a, P_\mu\rangle$  with total four momentum  $P_\mu$  or as

$$\chi(x_1 x_2; B) \equiv \langle 0 | T(\psi(x_1) \bar{\psi}^C(x_2)) | B \rangle$$

for the fermion-fermion bound state  $|B\rangle = |f f, P_\mu\rangle$ , where  $\psi^C$  stands for the charge conjugated field of  $\psi$ . Homogeneous Bethe-Salpeter equation for fermions in the ladder approximation is written in the following form

$$\chi(x_1 x_2; B) = -e^2 \int d^4 x_3 d^4 x_4 S_F(x_1 - x_3) \chi(x_3 x_4; B) S_F(x_2 - x_4) \gamma_\mu D_F^{\mu\nu}(x_3 - x_4) \gamma_\nu. \quad (2)$$

We can also write the BS equation in a differential form by applying Dirac operator;

$$(i\vec{\partial}_1 \cdot \gamma - m) \chi(x_1 x_2; B) (i\overleftarrow{\partial}_2 \cdot \gamma - m) = -e^2 D_F^{\mu\nu}(x_1 - x_2) \gamma_\mu \chi(x_1 x_2; B) \gamma_\nu, \quad (3)$$

where  $m$  is a dynamical mass and  $D_F$  is a photon propagator in the covariant gauge

$$D_F^{\mu\nu}(p) = \frac{g^{\mu\nu}}{p^2 + i\epsilon} + (\xi - 1) \frac{p_\mu p_\nu}{p^4}. \quad (4)$$

In momentum space we transform to center of mass and relative coordinate

$$X = (x_1 + x_2)/2, x = x_1 - x_2, \quad (5)$$

$$P = p_1 + p_2, p = (p_1 - p_2)/2. \quad (6)$$

Then we define  $\chi(P, p)$ , the Fourier transform of the Feynman amplitude, by

$$\chi(x_1 x_2; B) = \exp(iP \cdot X) \int d^4 p \exp(ip \cdot x) \chi(P, p). \quad (7)$$

The BS equation in momentum space assume the form

$$((\frac{P}{2} + p) \cdot \gamma - m) \chi(P, p) (- (\frac{P}{2} - p) \cdot \gamma - m) = - \frac{e^2}{(2\pi)^4} \int d^4 q D_F^{\mu\nu}(p - q) \gamma_\mu \chi(P, p) \gamma_\nu \quad (8)$$

for fa system. Since under charge conjugation vector particle is odd,  $\lambda$  should be replaced by  $-\lambda$ . For scalar case the  $\chi^S(P, p)$  is written in general which corresponds to spin singlet  $S(P, p)$  and triplet  $V(P, p)$  and tensor  $T(P, p)$

$$\chi^S(P, p) = S(P, p)I + P \cdot \gamma V^1(P, p) + p \cdot \gamma V^2(P, p) + \sigma_{\mu\nu} T(P, p) (P_\mu p_\nu - P_\nu p_\mu). \quad (9)$$

For total momentum  $P_\mu = 0$  case the BS amplitudes are decoupled to scalar-vector and tensor and we have a coupled equation for scalar-vector [2, 6, 7, 8]. If we substitute eq (9) to eq (8) we get a scalar-vector equation for Euclidean momentum  $p$

$$(m^2 - p^2)S(p^2) - 2p^2 V^2(p^2) = \lambda^S \int d^4 q \frac{S(q^2)}{(p - q)^2}, \quad (10)$$

and

$$(m^2 - p^2)p_\mu V^2(p^2) + 2p_\mu S(p^2) = \lambda^V \int d^4 q \frac{q_\mu V^2(q^2)}{(p - q)^2}. \quad (11)$$

$$\begin{aligned} \lambda^S &= (3 - \xi)\lambda \dots \text{for ff-system} \\ &= -(3 - \xi)\lambda \dots \text{for fa-system,} \end{aligned} \quad (12)$$

$$\lambda^V = 2\xi\lambda \dots \text{for ff-system} \quad (13)$$

$$= -2\xi\lambda \dots \text{for fa-system,} \quad (14)$$

where  $\lambda = e^2/(4\pi)^2$ . In the Landau gauge ( $\xi = 0$ )  $\lambda^V$  vanishes. The solution is given in ref[2]. Then eq (11) reduces to an algebraic equation and  $V^2(p^2)$  is obtained

$$V^2(p^2) = \frac{-2}{(m^2 - p^2)} S(p^2). \quad (15)$$

From eq (10) and (15) we obtain

$$\frac{(m^2 + p^2)^2}{m^2 - p^2} S(p^2) = \lambda^S \int d^4 q \frac{S(q^2)}{(p - q)^2}. \quad (16)$$

By using the formula(Klein-Gordon equation for photon)

$$\square_p \frac{1}{(p-q)^2} = -4\pi^2 \delta^{(4)}(p-q) \quad (17)$$

we get a differential equation for  $S(p^2)$  from eq (16)

$$(s \frac{d^2}{ds^2} + 2 \frac{d}{ds}) [\frac{(m^2 + s)^2}{m^2 - s} S(s)] = -\lambda^S S(s), \quad (18)$$

where  $s = p^2$ . The solution is characterized by hypergeometric function

$$\begin{aligned} S(s) &= \frac{m^2 - s}{(s + m^2)} (s + m^2)^{-1-\gamma/2} F(\alpha, \beta, \gamma; \frac{m^2}{m^2 + s}), \\ \alpha &= \frac{2 + \sqrt{1 + 4\lambda^S} - \sqrt{1 + 8\lambda^S}}{2}, \beta = \frac{2 + \sqrt{1 + 4\lambda^S} + \sqrt{1 + 8\lambda^S}}{2}, \\ \gamma &= 1 + \sqrt{1 + 4\lambda^S}, \end{aligned} \quad (19)$$

which satisfy the boundary condition

$$\begin{aligned} \lim_{s \rightarrow \infty} (s \frac{d}{ds} + 1) [\frac{(m^2 + s)^2}{m^2 - s} S(s)] &= 0, \\ \lim_{s \rightarrow 0} s^2 \frac{d}{ds} [\frac{(m^2 + s)^2}{m^2 - s} S(s)] &= 0. \end{aligned} \quad (20)$$

After angular integration eq (16) is rewritten

$$\frac{(m^2 + s)^2}{m^2 - s} S(s) = \frac{\lambda_S}{2s} \int_0^s ds' s' S(s') + \frac{\lambda_S}{2} \int_s^\infty ds' S(s'). \quad (21)$$

If we differentiate the above integral equation we get the boundary conditions. In the case of  $\alpha(\beta) = -n$ ,  $F$  is a  $n$ -th degree of hypergeometric series in  $s$ . We have a discrete set of spectrum if  $\lambda^S > 0$  (fermion-fermion system) and the eigenvalues are given

$$\lambda_n^S = (n+1)[3(n+1) + \sqrt{8n^2 + 16n + 9}], n = 0, 1, 2, \dots \quad (22)$$

For the lowest eigenvalue  $n = 0$  ( $\lambda_0 = \lambda_0^S/3 = 2$ ), we have

$$\chi_0(p) = \frac{m^2 - p^2 + 2p \cdot \gamma}{(m^2 + p^2)^5}. \quad (23)$$

For fermion-antifermion system there exists continuous spectrum for  $-1/8 \leq \lambda < 0$

$$S(s) = \frac{m^2 - s}{(s + m^2)} (s + m^2)^{-1-\gamma/2} F(\alpha, \beta; \alpha + \beta - \gamma + 1; \frac{s}{s + m^2}). \quad (24)$$

Here we show the profile of  $\chi_0(x)$  in FIG1. Fourier transformation of  $\chi(p)$  is defined

$$\chi(x) = \frac{1}{4\pi^2} \int p^3 dp \frac{J_1(px)}{px} \chi(p),$$

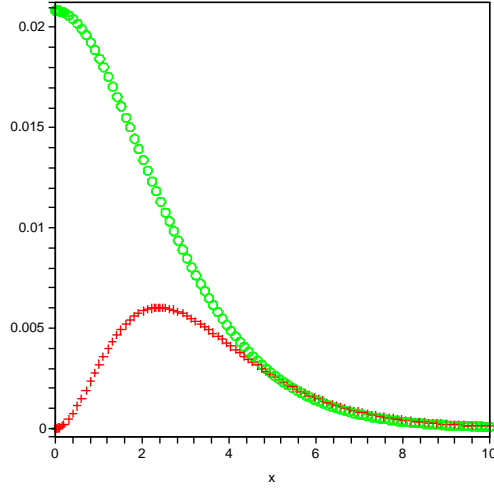


FIG. 1:  $4\pi^2\chi_0^S(x)(\times), 4\pi^2\chi_0^V(x)(\bigcirc)$  for  $m = 1$

$$\begin{aligned}\chi_{0S}(x) &= \frac{x^3}{768\pi^2}K_1(x), \\ \chi_{0V}(x) &= \frac{x}{1536\pi^2}(4xK_0(x) + (x^2 + 8)K_1(x)),\end{aligned}\tag{25}$$

for  $m = 1$ . In ref[4] introducing auxiliary field for composite operator effective action has been made. If we minimize it we can derive the Dyson-Schwinger equation for fermion and Bethe-Salpeter equation for boundstate. However for the first approximation to the lowest excitation, vector was neglected. Thus the BS equation for scalar meson is not a coupled equation with vector meson as eq(10),(11).

### B. Massless boson in QED<sub>3</sub>

In ref[3] three dimensional QED is considered as an effective model which describes the two dimensional superconductor, where gap has nodes and the low-energy fermionic excitations have linear dispersion. The fluctuating phase of superconductor is considered as Berry gauge fields which couples to spin degree of freedom of exciton. For these systems we assume the validity of relativistic treatment for fermion. For definiteness we use four-dimensional representation of  $\gamma$  matrix [5]. In ref[3] following type of equation for the eigenvalue  $e_n(|p|, m)$  and normalized eigenfunction  $\psi_n(r, |p|, m)$  for electron-hole system near the nodes were

discussed

$$[-\partial \cdot \partial - V_{eff}] \psi_n = e_n \psi_n, \quad (26)$$

$$\chi(p) = \frac{1}{16} \int d^3r \exp(-ip \cdot r) \sum_{ij} \langle \bar{\psi}_i(r) \psi_i(r) \bar{\psi}_j(0) \psi_j(0) \rangle, \quad (27)$$

$$\chi(p) = N \sum_n \frac{|\psi_n(0, |p|, m)|^2}{4e_n(|p|, m) + p^2}, \quad (28)$$

where  $V_{eff}$  is a potential by dressed gauge boson and has an invers-square form for small  $r$  and  $\chi(p)$  is a sum of boundstate propagator with residue  $|\psi_n|^2$  of each pole. The conclusion is that there are infinite numbers of negative energy. Here we simply notice the results

$$[-\partial \cdot \partial - \frac{\lambda}{16z^2}] \psi(z) = -\frac{1}{4} \psi, \lambda = (2 + \xi)g^2, \quad (29)$$

$$\psi(z) = \frac{K_{\sqrt{4-\lambda}/4}(z/2)}{\sqrt{z}}. \quad (30)$$

It is said that the continuum spectrum that is normalizable into a single boundstate would be a so called conformal tower

$$\psi_0(z) = \sqrt{\frac{\kappa^3}{2\pi}} \frac{K_0(\kappa|z|)}{\sqrt{\kappa|z|}}, \quad (31)$$

where  $-\kappa^2$  is the renormalized bound-state energy, and  $K_0$  is the modified Bessel function of the second kind. Apart from these realistic application to condensed matter physics it is interesting to solve the massless boundstate problems in three dimension. Now we return to relativistic BS equation for fa system to study the same problems. For  $P_\mu = 0$ ,  $T$  does not couple to  $S, V$ . We consider the equation as (16) in the previous section.

$$\frac{(m^2 + p^2)^2}{m^2 - p^2} S(p^2) = \lambda^S \int d^3q \frac{S(q^2)}{(p - q)^2}. \quad (32)$$

However we cannot solve the above equation as in four dimension since the identity is

$$\square_p \frac{1}{|p - q|} = -4\pi \delta^{(3)}(p - q) \quad (33)$$

in this case. Therefore we expand the left hand side of the eq (32) in  $p$  for large and small cases. We get

$$(-2m^2 - p^2)S(p) = \lambda^S \int d^3q \frac{S(q^2)}{(p - q)^2}, (m^2 \ll p^2), \quad (34)$$

$$(m^2 + 2p^2)S(p) = \lambda^S \int d^3q \frac{S(q^2)}{(p - q)^2}, (p^2 \ll m^2), \quad (35)$$

where  $\lambda^S = e^2/2\pi$ . We convert these equations in position space

$$(-2m^2 + \square_x)S_S(x) = \lambda^S \frac{S(x)_S}{|x|}, (m|x| \ll 1), \quad (36)$$

$$(m^2 - 2\square_x)S_L(x) = \lambda^S \frac{S(x)_L}{|x|}, (1 \ll m|x|). \quad (37)$$

First we solve the following equation for scalar-vector ff system  $S(x)$

$$(m^2 - \square_x)S(x) = -\lambda^S \frac{S(x)}{|x|}, \lambda^S = \frac{e^2}{2\pi},$$

$$\square_x = \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} + \frac{l(l+1)}{r^2}. \quad (38)$$

The relation among  $S(x)$ ,  $S_S(x)$ ,  $S_L(x)$  may be clear by scaling the mass or coupling constant. For simplicity we set  $m = 1$ . For the ground state  $l = 0$ , we have

$$\frac{d^2 S}{dr^2} + \frac{2}{r} \frac{d^2 S(r)}{dr^2} - S(r) - \frac{\lambda^S}{r} S(r) = 0. \quad (39)$$

For large  $r$  if we neglect terms which are proportional to  $1/r$ , we obtain

$$\frac{d^2 S(r)}{dr^2} = S(r), \quad (40)$$

from this  $S(r)$  behaves as  $e^{-r}$ . For small  $r$

$$S(r) = u(r)/r.$$

then the eq (39) becomes

$$\frac{d^2 u}{dr^2} + (-1 - \frac{\lambda}{r})u = 0. \quad (41)$$

Solutions of this equation must not diverge faster than the finite power of  $r$  and finite at  $r = 0$ . The solution which satisfies this latter condition is derived by Whittaker equation

$$\frac{d^2 W}{dz^2} + (-\frac{1}{4} - \frac{\lambda}{z} + \frac{1/4 - \mu^2}{z^2})W(z) = 0, \quad (42)$$

for  $\mu = 1/2$ , whose solution is expressed by linear combination of  $M_{\lambda, 1/2}$  and  $W_{\lambda, 1/2}$

$$S(x) = C_1 \frac{M_{-\lambda/2, 1/2}(2|x|)}{|x|} + C_2 \frac{W_{-\lambda/2, 1/2}(2|x|)}{|x|}. \quad (43)$$

$S(x)$  is expanded near origin

$$S(x)_{|x \rightarrow 0} = \frac{C_2}{|x| \Gamma(1 + \lambda/2)} + 2C_1 + .. \quad (44)$$



Therefore for normalizability of  $S(x)$  it is sufficient that  $1/\Gamma(1+\lambda/2)$  vanishes. This condition is realized for

$$1 + \lambda/2 = -n(n = 0, 1, 2, \dots). \quad (45)$$

This is an eigenvalue condition for the coupling constant  $\lambda$ . Thus fermion-antifermion(fa) system has discrete spectrum( $\lambda < 0$ ). In the case of positive coupling the function  $M$  diverges as  $\exp(x/2)$ [9]. Thus fermion-antifermion(ff) system has no bound states( $\lambda > 0$ ). Since the coupling constant has a dimension of mass, the mass of the ground state is largest for fixed coupling  $\lambda$ . Here we return to  $S_S(x)$  and  $S_L(x)$ . We have

$$S_S(x) = \frac{C_1 M_{-a,1/2}(2\sqrt{2}m|x|)}{|x|} + \frac{C_2 W_{-a,1/2}(2\sqrt{2}m|x|)}{|x|}, \quad (46)$$

$$S_L(x) = \frac{D_1 M_{a,1/2}(\sqrt{2}m|x|)}{|x|} + \frac{D_2 W_{a,1/2}(\sqrt{2}m|x|)}{|x|}, a = \frac{\sqrt{2}\lambda^S}{4m}. \quad (47)$$

Eigenvalues are given as

$$\frac{\sqrt{2}\lambda^S}{4m} = -n(n = 1, 2, \dots).$$

For the large distance in the case of these negative  $\lambda^S$ ,  $M_{a,1/2}(\sqrt{2}m|x|)$  blows up. Thus we choose  $D_1 = 0$ . Here we have an approximate solution of the integral equation for short and long distance

$$S(x) = \frac{C_1 M_{-a,1/2}(2\sqrt{2}m|x|)}{|x|} (m|x| \ll 1) \quad (48)$$

$$= \frac{D_2 W_{a,1/2}(\sqrt{2}m|x|)}{|x|} (1 \ll m|x|). \quad (49)$$

We choose  $C_1 = 1/2$  for normalization of  $S(x)$ . For small  $n$  we have an explicit form of  $M, W$  in terms of  $z = \sqrt{2}m|x|$

$$\begin{aligned} M_{1,1/2}(z)/z &= \exp(-z)2, \\ M_{2,1/2}(z)/z &= \exp(-z)2(1-z), (z \ll 1). \end{aligned} \quad (50)$$

$$\begin{aligned} W_{-1,1/2}(z)/z &\simeq \exp(-z)/2z^2, \\ W_{-2,1/2}(z)/z &\simeq \exp(-z)/z^3, (1 \ll z). \end{aligned}$$

Since at long distance the function  $W$  strongly damps, we cut-off long distance and the solution may be approximated

$$S(z)_n = M_{\lambda,1/2}(2|z|)/2|z|, \lambda = n. \quad (51)$$

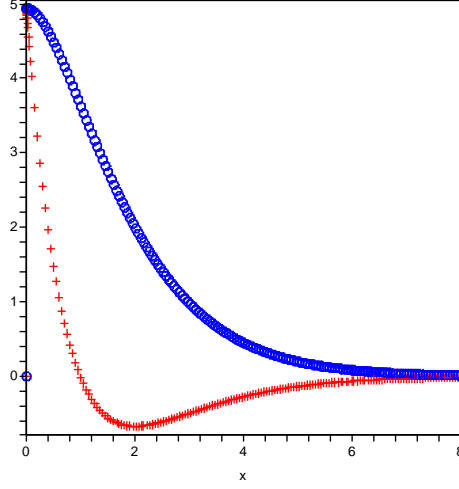


FIG. 2:  $\chi_2^S(x)(\times), \chi_2^V(x)(\square)$  for  $m = 1$

If we transform into momentum space we get for  $n = 1, 2, \sqrt{2}m = 1$

$$\begin{aligned}\chi_1(p^2) &= \int x^2 d|x| \frac{\sin(p|x|)}{(p|x|)} S(|x|)_1 = \frac{1}{(p^2 + 1)^2} \left(1 - \frac{2\gamma \cdot p}{1 - p^2}\right), \\ \chi_2(p^2) &= \frac{16\pi(-1 + p^2 + 2p \cdot \gamma)}{(p^2 + 1)^3}.\end{aligned}\quad (52)$$

Here we notice that the vector part of  $\chi_1$  is not normalizable, therefore we avoid  $n = 1$  case for scalar-vector fa systems. Thus the ground state must be given by  $n = 2$ . Correct solution in the whole region may be evaluated by numerical analysis of integral equation (32) with angular intergral

$$\frac{(m^2 + p^2)^2}{m^2 - p^2} S(p) = \frac{\lambda}{(2\pi)^2 p} \int_0^\infty q dq S(q) \ln\left(\frac{p+q}{|p-q|}\right), \frac{\lambda\sqrt{2}}{4m} = 2, \quad (53)$$

with an input of Fourier transform of eq (49)

$$S_S(p) = \frac{16(-2m^2 + p^2)}{(p^2 + 2m^2)^3} \quad (54)$$

in the right hand side of eq (53). We see the profile of  $\chi_2(x)$  in FIG2.

### III. WARD-TAKAHASHI-IDENTITY FOR AXIALVECTOR CURRENTS

In this section we examine the Ward-Takahashi-identity for the axialvector currents. In our approximation to BS equation fermion mass is assumed to be dynamical. Therefore if

we have a solution of the BS equation for psedoscalar it must satisfy the Ward-Takahashi-identity for the conservation of axialvector currents[13]

$$\lim_{q \rightarrow 0} q_\mu S_F(p') \Gamma_{5\mu}(p', p) S_F(p) = \{S_F(p), \gamma_5\} \neq 0, \quad (55)$$

where  $S_F(p)$  is a solution of the Dyson-Schwinger equation or non-perturbative solution of the fermion propagator. The vertex function  $\Gamma_{5\mu}$  has a massless pole  $q_\mu/q^2 \chi^P(q)$ , where  $\chi^P(q)$  is a BS amplitude of psedoscalar Goldstone boson as a consequence of chiral symmetry breaking. Following the notation in ref[5],  $U(2)$  chiral symmetry is generated by  $\{I, \gamma_4, \gamma_5, \gamma_{45}\}$  which is broken by dynamical fermion mass to a  $U(1) \times U(1)$  symmetry generated by  $\{I, \gamma_{45}\}$  for the degree of freedom of scalar and psedoscalar as  $\{\sigma, \pi\}$ . The set of  $\gamma$  matrices is

$$\gamma_0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}, \gamma_{1,2} = \begin{pmatrix} \sigma_{1,2} & 0 \\ 0 & -\sigma_{1,2} \end{pmatrix}, \gamma_4 = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \gamma_5 = \begin{pmatrix} 0 & -iI \\ iI & 0 \end{pmatrix},$$

$$\gamma_{45} = -i\gamma_4\gamma_5, \{\gamma_\mu, \gamma_4\} = 0, \{\gamma_\mu, \gamma_5\} = 0 \quad (56)$$

Currents  $\{\gamma_{\mu 4}, \gamma_{\mu 5}\}$  are a analog of axialvector currents and have a doublet of Goldstone boson which is called axial-scalar,

$$\begin{pmatrix} \chi^{(4)}(P, q) \\ \chi^{(5)}(P, q) \end{pmatrix}^{AS} = \begin{pmatrix} \gamma_4 \\ \gamma_5 \end{pmatrix} S + \begin{pmatrix} \gamma_{\mu 4} \\ \gamma_{\mu 5} \end{pmatrix} (V^1 P_\mu + V^2 q_\mu) + \epsilon_{\mu\nu\rho} P^\mu q^\nu \begin{pmatrix} \gamma^{\rho 5} \\ -\gamma^{\rho 4} \end{pmatrix} T. \quad (57)$$

Scalar-PS( $\gamma_{45}$ ) BS amplitude is

$$\chi^S(P, q) = S + \gamma \cdot q V^2 + \gamma \cdot P V^1 + \epsilon_{\mu\nu\rho} P^\mu q^\nu \gamma^{\rho 45} T. \quad (58)$$

$$\chi^{PS}(P, q) = S(P, q) \gamma_{45} + q^\mu \gamma_{\mu 45} V^2 + P^\mu \gamma_{\mu 45} V^1 + \epsilon_{\mu\nu\rho} P^\mu q^\nu \gamma^{\rho 45} T. \quad (59)$$

To check the Ward-Takahashi-identity we consider the equation for axial-scalar

$$(m^2 + p^2) \chi^{AS}(p) = \frac{\lambda^{AS}}{(2\pi)^3} \int d^3 p' \frac{\chi^{AS}(p')}{(p - p')^2}, \quad (60)$$

$$(m^2 - \square_x) \chi^{AS}(x) = \lambda^{AS} \frac{\chi^{AS}(x)}{|x|}, \lambda^{AS} = \lambda. \quad (61)$$

for ff system. Sign of the coupling is opposite to the one in even dimension. This is the same equation discussed in scalar-vector case and we have solutions

$$\chi^{AS}(x) = \frac{M_{\lambda/2m, 1/2}(2m|x|)}{|x|},$$

$$\chi_1^{AS}(p) = \frac{1}{(p^2 + m^2)^2}. \quad (62)$$

for  $\lambda/2m = n(1, 2, \dots)$ . Following the Ward-Takahashi-identity  $\chi^{AS}(x)$  for  $\text{fa}$  is derived from the scalar part of the fermion propagator. Dyson-Schwinger equation for fermion propagator is written

$$\Sigma(p) = \frac{e^2}{(2\pi)^3} \int d^3k \gamma_\mu \frac{k \cdot \gamma + M(k)}{k^2 + m^2} \gamma_\nu \frac{g_{\mu\nu} - (p-k)_\mu(p-k)_\nu/(p-k)^2}{(p-k)^2}, \quad (63)$$

$$M(p) = \frac{e^2}{4\pi^2} \int d^3k \frac{M(k)}{k^2 + m^2} \frac{1}{(p-k)^2} \quad (64)$$

in the Landau gauge with linear approximation  $m = M(0)$ . We define  $F(x)$  as the Fourier transformation of  $M(p)/(p^2 + m^2)$

$$F(x) = \int \frac{d^3p}{(2\pi)^3} \exp(ip \cdot x) \frac{M(p)}{p^2 + m^2}, \quad (65)$$

$$\begin{aligned} (-\square_x + m^2)F(x) &= \frac{2e^2}{(2\pi)^6} \int d^3p e^{ip \cdot x} \int d^3p' \frac{M(p')}{p'^2 + m^2} \frac{1}{k^2} \\ &= 2e^2 \int \frac{d^3p'}{(2\pi)^3} e^{ip' \cdot x} \frac{M(p')}{p'^2 + m^2} \int \frac{d^3k}{(2\pi)^3} \frac{e^{ik \cdot x}}{k^2} \\ &= 2e^2 \frac{1}{4\pi |x|} F(x), \end{aligned} \quad (66)$$

where  $k = p' - p$  and used that  $\int d^3p = \int d^3k$ . We arrive at a Schrödinger like equation for  $F(x)$

$$(-\square_x + m^2)F(x) = \lambda \frac{F(x)}{|x|}, \quad \lambda = \frac{e^2}{2\pi}. \quad (67)$$

Its solution is given as the groundstate of hydrogen atom

$$F(x) = \frac{m^2}{8\pi} \exp(-m|x|), \quad F(p) = \frac{m^3}{(p^2 + m^2)^2}. \quad (68)$$

In this case the ground state solution corresponds to  $\lambda/2m = 1$  ( $m = e^2/4\pi$ ), where mass is largest for fixed coupling[10]. We also meet this condition in the analysis of the fermion propagator based on low-energy theorem[11,12]. In fact we can determine a precise infrared behaviour of the propagator in the above analysis but do not have a definite ultraviolet behaviour. However if we demand the nonvanishment of the order parameter  $\langle \bar{\psi}\psi \rangle \neq 0$ , we obtain the condition for the anomalous dimension of the wave function which must be unity. This condition is approximately satisfied in the case of  $1/N$  correction for photon propagator with vanishing bare mass in the Landau gauge[12]. This is just the statement of Nambu-Goldstone theorem[14,15].

#### IV. SUMMARY

In this work we studied the solutions of spinor-spinor Bethe-Salpeter equation for massless boson in QED<sub>3</sub> with ladder approximation in the Landau gauge. It is not easy to solve this equation directly in three dimension. However we use the approximate form of the integral equation for large and small momentum respectively. These are converted to the differential equation which are similar to the Schrödinger type equation. In this case an eigenvalue are determined by boundary condition at  $x = 0$ . Thus we obtain solutions for massless scalar-vector fa systems with discrete spectrum. In our approximation massless scalar-vector ff system does not seem to exist in three dimension. Finally it is shown that Ward-Takahashi-identity for axial currents is satisfied with the Dyson-Schwinger equation for the fermion propagator in the linear approximation to dynamical mass .

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